Singular Points Detection Based on Zero-Pole Model in Fingerprint Images

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Abstract—An algorithm is proposed, which combines Zero-pole Model and Hough Transform (HT) to detect singular points. Orientation of singular points is defined on the basis of the Zero-pole Model, which can further explain the practicability of Zero-pole Model. Contrary to orientation field generation, detection of singular points is simplified to determine the parameters of the Zero-pole Model. HT uses rather global information of fingerprint images to detect singular points. This makes our algorithm more robust to noise than methods that only use local information. As the Zero-pole Model may have a little warp from actual fingerprint orientation field, Poincare index is used to make position adjustment in neighborhood of the detected candidate singular points. Experimental results show that our algorithm performs well and fast enough for real-time application in database NIST-4.

Index Terms—Zero-pole model, Hough Transform(HT), orientation field, singular points.

1 INTRODUCTION

Singular points are viewed as global feature of fingerprint images and invariant to translation, rotation, enlargement, and shrinking. Singular points can be used in fingerprint pattern classification [1], [2], [6] to reduce search space in large database and as reference points in fingerprint minutiae matching [3], [4]. Srinivasan and Murthy [11] use local orientation histogram to detect singular points. Where local orientation histogram does not exhibit a well-pronounced peak is likely to be a singular region. Zheng et al. [22] and Koo and Kot [23] use curvature-based method to detect singular points. High-curvature regions are denoted as singular regions. Nilsson and Bigun [25], [26], [27] (also called NB method) use two complex filters tuned to detect singular points in multiple resolution scale. Where the magnitude of complex filter response is high, the position is retained as a singular region. Kawagoe and Tojo [19] proposed Poincare index to detect singular points. Poincare index is always the most classical way to detect singular points and used widely [2], [5], [6], [8], [19]. However, Poincare index only uses local information to detect singular points and easy to be affected by noise. Singular points are core and delta, as illustrated in Fig. 1.

Sherlock and Monro [7] proposed a Zero-pole Model, which can describe fingerprint orientation field. Zero-pole Model reveals that the orientation at a point is determined by the number and positions of singular points plus a constant correction term. The parameters of the Zero-pole Model are all the singular points. Near to singular points, the Zero-pole Model can describe orientation field exactly. However, far from singular points, the orientation field described by the Zero-pole Model may have a little warp from actual orientation field. Despite some flaws, the Zero-pole Model is valid for a whole fingerprint image. Vizcaya and Gerhardt [8] present a nonlinear orientation field model to ameliorate the Zero-pole Model. In order to improve the accuracy of the Zero-pole Model, Zhou and Gu proposed another model-based method for the estimation of orientation field [13], [14], [15], [24]. Cappelli et al. [9], [16] use the Zero-pole Model to synthesize fingerprint images as a commercial database. All these methods use a model to generate a fingerprint orientation field. Contrary to orientation field generation, our work is to determine the parameters of the Zero-pole Model, i.e., detection of singular points. All these improved models are difficult to compute compared with the Zero-pole Model. The Zero-pole Model is ideal to describe the topology of fingerprint image. Especially, it can well explain why the delta is a confluent point of three different flow-like directions. This can further prove the practicability of the Zero-pole Model. Far from singular points, as the Zero-pole Model may have a little warp from actual orientation field, singular points detected by the Zero-pole Model may have a little warp from their real positions. Therefore, Poincare index is used to make position adjustment in the neighborhood of the detected candidate singular points. For above reasons, Zero-pole Model is appropriate to be the basis of our work.

The approach proposed in this paper combines Zero-pole Model and Hough Transform (HT) to detect singular points. The orientation of singular points is defined on the basis of the Zero-pole Model, which can further explain the practicability of the Zero-pole Model. The detection of singular points is to determine the parameters of the Zero-pole Model. Methods only using local information to detect singular points are prone to be affected by noise and result in a lot of false
responses. Consequently, much work is needed to remove the false responses. HT is useful for the detection of curves such as circles and ellipses [10], [17]. In our work, HT uses rather global information of fingerprint images to detect singular points. This makes our algorithm more robust to noise than methods which only use local information. Finally, Poincare index is used to refine positions of the candidate singular points. In a way, the results of singular points detection will be limited by the accuracy of Poincare index. In fact, global information combined with local information have good performance in singular points detection. NB method also uses global information to detect singular points and local information to refine the positions of singular points. NB method and our work all use both global information and local information to detect singular points. However, a detailed algorithm to realize singular point detection is totally different. The NB method uses two complex filters to make a convolution with fingerprint tensor field. According to their complex filter response, the argument is the estimated orientation at the point. What’s more, the larger the magnitude, the bigger the probability that there is a singular point at the position. However, our work uses HT to determine the parameters of the Zero-pole Model.

In the following sections, singular points detection algorithm will be presented in detail. Section 2 describes fingerprint image preprocessing. Section 3 gives the orientation definition of singular points and presents the algorithm to detect singular points. Section 4 shows our experimental results tested on simulated fingerprint orientation field and NIST-4. Finally, Section 5 gives the conclusion of this paper.

2 PREPROCESSING OF FINGERPRINT IMAGES

Let $I$ be a gray-level image of a fingerprint, and $I(i,j)$ represent the gray value of pixel $(i,j)$. $Gx(i,j)$ and $Gy(i,j)$ are the gradients at the horizontal axis and vertical axis, respectively. Let $W_c$ be a block centered at pixel $(i,j)$, and the block size is $w \times w$. Let

$$dy(i,j) = \sum_{u=-\frac{w}{2}}^{\frac{w}{2}} \sum_{v=-\frac{w}{2}}^{\frac{w}{2}} 2 \times Gx(u,v) \times Gx(u,v),$$

(1)

$$dx(i,j) = \sum_{u=-\frac{w}{2}}^{\frac{w}{2}} \sum_{v=-\frac{w}{2}}^{\frac{w}{2}} \left(Gx(u,v)^2 - Gy(u,v)^2\right).$$

(2)

The local ridge orientation of $(i,j)$ is perpendicular to its gradient vector. According to methods in [5] and [18], the coarse local ridge orientation of pixel $(i,j)$ is

$$\theta(i,j) = \frac{1}{2} \times \tan^{-1}\frac{dy(i,j)}{dx(i,j)} + \frac{\pi}{2},$$

(3)

A note must be pointed out: on the pixel level, if both $Gx(i,j)$ and $Gy(i,j)$ are very small, it means that there is no orientation; if $Gx(i,j)$ is large and $Gy(i,j)$ is very small, it means that there is an $x$-direction only; if $Gy(i,j)$ is large and $Gx(i,j)$ is very small, it means that there is a $y$-direction only. Methods in [18], [28] are used to segment fingerprint into foreground, background, and noisy region. A variance of gray levels is computed in directions along ridges and orthogonal to the orientation of ridges. If the variance in both directions has a low value, the pixel $(i,j)$ is in the background. If there is no orientation, the pixel $(i,j)$ is also assigned to the background. In a nonbackground region, if the variance in the direction orthogonal to ridge orientation is high, and the variance along the ridge is low, the pixel $(i,j)$ is in the foreground. Else, other pixels are in the noisy region.

Coarse orientation $\theta(i,j)$ may be influenced by noise in fingerprint images. Orientation histogram averaging is used to smooth the coarse orientation field in block $W_c$ centered at pixel $(i,j)$. Assume that the orientation histogram peak $\theta_1$ is $n_1$ times. The rest may be deduced by analogy, the orientation $\theta_2$ is $n_2$ times, the orientation $\theta_3$ is $n_3$ times, and so on. In order to decrease the affection of noise, the smoothed orientation of the pixel $(i,j)$ is

$$\theta(i,j) = \frac{\theta_1 \times n_1 + \theta_2 \times n_2 + \ldots + \theta_j \times n_j}{n_1 + n_2 + \ldots + n_j},$$

(4)

where $|\theta_1 - \theta| < T_\theta$, $T_\theta$ is a predefined experiential threshold.

In the above segmentation, there may be isolated pixels, i.e., a pixel in the background or noisy region is surrounded by the foreground, and vice versa. During smoothness, isolated pixels are removed. In this way, the fingerprint is segmented into the foreground, background, and noisy region.

In a real fingerprint, our algorithm will work on blocks to detect singular points. If the block size is $b \times b$, block orientation is the dominant orientation in each block computed by orientation histogram.

3 BASED ON ZERO-POLE MODEL TO DETECT SINGULAR POINTS

3.1 Zero-Pole Model

Sherlock and Monro [7] proposed Zero-pole Model for orientation field estimation. In the Zero-pole Model, core is considered as zero, and delta is considered as pole in the complex plane. The orientation $o(z)$ of a point $z$ in the fingerprint is an argument of complex function $p(z)$. $p(z)$ and $o(z)$ are defined as follows:
In the above formulas, \( z_{ci} \) and \( z_{dj} \) are the \( i \)th core and \( j \)th delta of fingerprint, respectively, and \( o_\infty \) is a constant correction term. At the point \( z \), the influence of core \( z_{ci} \) is 
\[ \frac{1}{2} \times \arg(z - z_{ci}), \]
and delta \( z_{dj} \) is 
\[ -\frac{1}{2} \times \arg(z - z_{dj}). \]
Fig. 2 shows the orientation field of a core and a delta generated by the Zero-pole Model. Orientation at the point \( z \) is the sum of influence of all cores and deltas. According to the knowledge of complex function, (6) can be rewritten as

\[
o(z) = \left[ o_\infty + \frac{1}{2} \times \left( \sum_{i=1}^{K} \arg(z - z_{ci}) \right) - \sum_{j=1}^{L} \arg(z - z_{dj}) \right] \mod \pi.
\]  

(7)

Zero-pole Model is an ideal orientation field model to describe the topology of fingerprint images. It can be used to calculate the orientation of a point and reconstruct a noisy image [8].

### 3.2 Orientation Definition of Singular Points

Zero-pole Model can define the orientation of singular points. Especially, it can well explain why delta is a confluent point of three different flowlike directions. That means, the Zero-pole Model describes topology of fingerprint reasonably. This can further prove the feasibility of the Zero-pole Model to be the basis of our work.

Core is the end point of the innermost curve ridge, and delta is the confluent point of three different flowlike directions [5]. In a fingerprint image, the orientation of a point is its tangent orientation along the ridge that the point belongs to.

The orientation of the \( n \)th core \( c_m \) is its tangent orientation

\[
o(c_m) = \arg(z - c_m),
\]  

(8)

where \( z \) is a point on the tangent of the \( n \)th core \( c_m \). In the Zero-pole Model, the orientation of a point is given by (7). Namely, the orientation of the \( n \)th core \( c_m \) is

\[
o(c_m) = \left[ \frac{1}{2} \times o(c_m) + \frac{1}{2} \times \left( \sum_{i=1, i \neq m}^{K} \arg(c_m - z_{ci}) \right) - \sum_{j=1}^{L} \arg(c_m - z_{dj}) \right] \mod \pi.
\]  

(9)

Formula (9) is equal to

\[
o(c_m) = \left[ 2 \times o_\infty + \left( \sum_{i=1, i \neq m}^{K} \arg(c_m - z_{ci}) \right) - \sum_{j=1}^{L} \arg(c_m - z_{dj}) \right] \mod \pi.
\]  

(10)

From (10), it can be seen that orientation of a core is affected by correction term \( o_\infty \) and all other singular points.

The same with the way to define orientation of core, for the \( n \)th delta \( d_n \), its tangent orientation is

\[
o(d_n) = \arg(z - d_n),
\]  

(11)

where \( z \) is a point on the tangent of the \( n \)th delta \( d_n \). According to (7),

\[
o(d_n) = \left[ -\frac{1}{2} \times o(d_n) - \frac{1}{2} \times \left( \sum_{j=1, j \neq n}^{L} \arg(d_n - z_{dj}) \right) - \sum_{i=1}^{K} \arg(d_n - z_{ci}) \right] \mod \pi.
\]  

(12)
Formula (12) is equal to
\[
o(z_dn) = \frac{\sum_{i=1}^{K} \arg(z_{dn} - z_{oi}) - \sum_{j=1, j\neq n}^{L} \arg(z_{dn} - z_{dj})}{3} + \frac{2 \times o_{\infty} + 2k\pi}{3}, \quad k = 0, 1, 2.
\] (13)

From (13), \(o(z_dn)\) has three values, and the difference among the three values is \(\pi\). This can explain why delta is the confluent point of three different flowlike directions.

### 3.3 HT to Detect Core

The parameters of the Zero-pole Model are the number and positions of singular points. In fact, each singular point has a mainly affected neighboring area \(N\). In other words, orientation field in area \(N\) is mainly determined by its closest singular point, while all other singular points just have a constant effect on \(N\). Singular points detection is to determine the parameters of the Zero-pole Model, but (7) is very difficult to be directly computed. HT will be used to detect singular points based on the affected neighboring area \(N\) and orientation field computed in Section 2. HT is easy to compute and uses rather global information to detect singular points. This makes our algorithm more robust to noise than methods which only use local information. In this way, all the singular points can be detected. For the modified Zero-pole Model, HT can also be applicable to detect singular points.

A core in fingerprint labeled as \(z_c\) in complex plane, orientation \(o_{z_c}\) defined by Zero-pole Model at a point \(z_1\) is
\[
o_{z_c} = \left[ o_{\infty} + \frac{1}{2} \times \arg(z_1 - z_c) \right] \mod \pi.
\] (14)

Orientation \(o_{z_c}\) at a point \(z_2\) is
\[
o_{z_c} = \left[ o_{\infty} + \frac{1}{2} \times \arg(z_2 - z_c) \right] \mod \pi.
\] (15)

Formula (15) subtracts (14), we can get that
\[
o_{z_c} - o_{z_c} = \frac{1}{2} \times [\arg(z_2 - z_c) - \arg(z_1 - z_c)].
\] (16)

Formula (16) is equal to
\[
\arg(z_2 - z_c) - \arg(z_1 - z_c) = 2 \times (o_{z_c} - o_{z_c}).
\] (17)

Assume \(o = 2 \times (o_{z_c} - o_{z_c})\), \(o_{z_c}\), and \(o_{z_c}\) can be obtained from Section 2. So that the value of \(o\) can be computed. In (17), core \(z_c\) is the point that argument difference between vector \(z_c - z_1\), and \(z_c - z_2\) is constant \(o\).

Formula (17) is equal to
\[
\arg(z_2 - z_c) = o + \arg(z_1 - z_c).
\] (18)

Assume \(o_1 = \arg(z_1 - z_c)\), then
\[
z_c - z_1 = r_1 \times e^{i(o_1 + \pi)},
\] (19)
\[
z_c - z_2 = r_2 \times e^{i(o_1 + o + \pi)},
\] (20)

In the above two formulas, \(r_1\) is the complex modulus of vector \(z_c - z_1\), and \(r_2\) is the complex modulus of vector \(z_c - z_2\). As orientation of fingerprint is in the range \([0, \pi]\) [12], (19) and (20) are equal to
\[
z_c - z_1 = r_1 \times e^{i\alpha},
\] (21)
\[
z_c - z_2 = r_2 \times e^{i(\alpha + o)}.\] (22)

Formula (22) subtracts (21), we can get that
\[
(z_1 - z_2) \times e^{-i\alpha} = r_2 \times e^{i\alpha} - r_1.
\] (23)

In (23), the real part on the left is equal to the real part on the right, and the imaginary part on the left is equal to the imaginary part on the right. From the above formulas, finally, we can get that
\[
x_c = x_1 + r_1 \times \cos(o),
\] (24)
\[
y_c = y_1 + r_1 \times \sin(o),
\] (25)
\[
r_1 = \cos(o) \times \left[ \frac{\cos(o)}{\sin(o)} \times (y_1 - y_2) - (x_1 - x_2) \right]
\]
\[
+ \sin(o) \times \left[ \frac{-\cos(o)}{\sin(o)} \times (x_1 - x_2) - (y_1 - y_2) \right].
\] (26)

where \((x_c, y_c), (x_1, y_1),\) and \((x_2, y_2)\) are coordinates of \(z_c, z_1,\) and \(z_2\) in a Cartesian plane, respectively. Especially, when \(z_1\) and \(z_2\) are very close, \(z_c\) will be in approximate superposition with \(z_1\) and \(z_2\). Formulas (24), (25), and (26) determine the curve of core \(z_c\).

When two points \(z_1\) and \(z_2\) are chosen from fingerprint, their coordinates \((x_1, y_1)\) and \((x_2, y_2)\) are given. Their orientation \(o_{z_1}\) and \(o_{z_2}\) can be obtained from Section 2, so that the value of \(o = 2 \times (o_{z_2} - o_{z_1})\) can be computed. The argument \(o_1\) of vector \(z_c - z_1\) is the parameter of (24), (25), and (26). When \(o_1\) changes in orientation range \([0, \pi]\), the curve of core can be drawn. Fig. 3 shows the curve of core, also called Hough curve. The coordinates of \(z_1\) and \(z_2\) are \((x_1, y_1)\) and \((x_2, y_2)\) labeled as “\(x, y\)” \(o_{z_1}\) and \(o_{z_2}\) are their orientation obtained from Section 2, \(o = 2 \times (o_{z_2} - o_{z_1})\). Fig. 3a shows an example of \(o = \pi/3\). Fig. 3b shows examples of a set of values of \(o\). Fig. 4 simulates Hough curves of three points passing through core. \((x_c, y_c)\) is coordinate of core \(z_c\), labeled as “\(x, y\)” \((x_1, y_1), (x_2, y_2),\) and \((x_3, y_3)\) are coordinates of three points \(z_1, z_2,\) and \(z_3\) labeled as “\(x, y\)” \(o_{z_1}, o_{z_2},\) and \(o_{z_3}\) are their orientation obtained from Section 2. \(o_{z_2} = 2 \times (o_{z_2} - o_{z_1}), \quad o_{z_3} = 2 \times (o_{z_2} - o_{z_3}),\) and \(o_{z_2} = 2 \times (o_{z_2} - o_{z_3}).\)

### 3.4 HT to Detect Delta

The same with the way to detect core, a delta in fingerprint labeled as \(z_d\) in complex plane, orientation \(o_{z_d}\) defined by the Zero-pole Model at a point \(z_1\) is
\[
o_{z_d} = \left[ o_{\infty} - \frac{1}{2} \times \arg(z_1 - z_d) \right] \mod \pi.
\] (27)
In above formulas, \( \pi \) determine the curve of delta as \( \pi/5 \). (a) \( o = 2 \times (o - o_1) = \pi/3 \). (b) Hough Curves comparison with different \( o = 2 \times (o - o_3) \).

The orientation \( o \) at a point \( z \) is

\[
o = o_{\infty} - \frac{1}{2} \times \arg(z) \mod \pi. \tag{28}
\]

Formula (28) subtracts (27), we can get that

\[
o - o_1 = -\frac{1}{2} \times \left[ \arg(z) - \arg(z_1) \right]. \tag{29}
\]

Assume \( o_1 = \arg(z_1 - z_d) \) and \( o = 2 \times (o_2 - o_1) \), the same computation with the way to detect core, we can get that

\[
x_d = x_1 + r_1 \times \cos(o_1), \tag{30}
\]

\[
y_d = y_1 + r_1 \times \sin(o_1), \tag{31}
\]

\[
r_1 = \cos(o_1) \times \left[ -\sin(o_1) \times (y_1 - y_2) - (x_1 - x_2) \right] + \sin(o_1) \times \left[ \cos(o_1) \times (x_1 - x_2) - (y_1 - y_2) \right]. \tag{32}
\]

In above formulas, \((x_d, y_d)\), \((x_1, y_1)\), and \((x_2, y_2)\) are coordinates of \( z_d, z_1 \), and \( z_2 \) in the Cartesian plane, respectively. \( r_1 \) is the complex modulus of vector \( z_d z_1 \). Formulas (30), (31), and (32) determine the curve of delta \( z_d \).

### 3.5 Poincare Index and Singular Points

At present, Poincare index is the most classical way to identify singular points [2], [5], [6], [8], [19]. Poincare index is the complete rotations of orientation along a closed curve around a point. For fingerprint images, the Poincare index \( Pin(x, y) \) is characterized as follows:

\[
Pin(x, y) = \begin{cases} 
\pi & \text{if } (x, y) \text{ is a core,} \\
-\pi & \text{if } (x, y) \text{ is a delta,} \\
0 & \text{others.}
\end{cases} \tag{33}
\]

In the computed orientation field of a fingerprint, following along a counterclockwise closed curve around a core and adding the differences between subsequent orientations, it will result in a cumulative orientation change \( \pi \) [5]. Carrying out this procedure around a delta, it will result in \(-\pi \) [5]. According to Galton-Henry fingerprint classification rules [20], [21], arch has no singular point. However, in the Zero-pole Model, arch has one core and one delta whose distance is very close, and their influence on orientation field counteracts each other. In arch fingerprint, HT will also detect a core and a delta on the basis of the Zero-pole Model. However, cumulative orientation change around the detected core and delta will be zero. If the detected core and delta by HT are very close and the Poincare index of them are both zero, it is classified into arch and the detected singular points are erased.

As the Zero-pole Model may have a little warp from the actual orientation field far from singular points, the detected candidate singular points by HT may have a little warp from their real positions. Poincare index is necessary to refine the positions of the candidate singular points.
4 EXPERIMENTAL RESULTS

In this section, two experiments are carried out to test the performance of our algorithm. First, our algorithm is applied to a simulated fingerprint orientation field generated by the Zero-pole Model. In the second experiment, our algorithm is used to detect singular points in real fingerprint images.

4.1 HT to Detect Singular Points in Simulated Fingerprint Orientation Field

This experiment is to validate the idea that HT can successfully detect singular points. Here, any fingerprint models can be used to simulate an orientation field. For convenience, the Zero-pole Model is used to simulate an orientation field. The parameters of the Zero-pole Model, number and positions of singular points, are first set. Simulated orientation field is generated by taking the number and positions of singular points as inputs of (7). Fig. 5 shows the simulated orientation field. The simulated fingerprint orientation field is consistent with the Zero-pole Model. The accuracy is perfect using HT to detect singular points on this condition. Poincare index is not necessary to refine positions of the detected singular points.

The simulated orientation field is divided into nonoverlapping blocks with size 4 × 4. In this experiment, block size is chosen randomly, and it can also be 8 × 8, 16 × 16, or something else. In HT, the search pattern is taking \(z_1\) and \(z_2\) as \((i, j)\) and \((i + 1, j)\), \((i, j)\) and \((i, j + 1)\), \((i, j)\) and \((i + 1, j + 1)\), \((i + 1, j)\) and \((i, j + 1)\), respectively. This kind of search pattern can fully use fingerprint information. Fig. 6 shows the search pattern. The HT search pattern is done for all the blocks in the simulated orientation field. In HT, each block is an accumulator cell, and the number that the curves pass through each block is accumulated. For visual aid, the number in all blocks is stretched to interval \([0, 255]\) gray levels to form a gray image. The block that has much larger gray value than others corresponds to a singular point.

Figs. 7 and 8 show the simulated orientation field upon the obtained gray images. In Fig. 8, two blocks have much larger gray value than others and correspond to two singular points. In Fig. 8a, the detected upper core has a larger gray value than the lower one. This is because each singular point has its main affected area, and the area size is different from each other. The larger the affected area size, the more HT curves pass through the singular point, i.e., a larger gray value.

4.2 HT to Detect Singular Points in Real Fingerprint Images

This experiment is carried out on database NIST-4, and image size is 512 × 512. Many fingerprint images in NIST-4 are suffering from creases, scars, and smudges. Our algorithm is implemented with VC++ on a 2.80 GHz PC computer. The average total time just for singular points detection is 0.165 second, not including the time for fingerprint image preprocessing. In part A, as simulated orientation field is consistent with the Zero-pole Model, the detected singular points by HT are located precisely. However, for real fingerprint images, far from singular points, the orientation field generated by Zero-pole Model may have a little warp from actual orientation field. Therefore, singular points detected by HT may have a little warp from their real positions. Poincare index is necessary to refine the positions of the candidate singular points. In Section 2, fingerprint image has been segmented into foreground, background, and noisy region. In this section, HT search pattern only works on foreground.
Fingerprint image is divided into blocks, and the block size is 16 × 16 because the average ridge width of fingerprint is about 16 pixels. The block direction is the dominant orientation computed by orientation histogram in each block. Each block in fingerprint image is an accumulator cell during HT. However, the HT search pattern is only applied to blocks in the foreground. The same way with part A, a gray-value image can be obtained after HT. According to Henry classification rules [21], arch has no singular point, tent arch or loop has one pair of singular points, and whorl or twin has two pairs of singular points. In theory, HT should detect two pairs of singular points for whorl or twin, as Figs. 10b and 10d show. Sometimes, the second large gray-value block is a result from noise. For a lot of whorls or twins, their two cores are fairly close. The two cores detected by HT are so close that they cannot be distinguished from each other, as Fig. 11b shows. For the above reasons, only the block that has the maximum gray value is considered to be a candidate singular point. Hence, a candidate core and delta can be found by HT. Then, Poincare index is used to refine the positions of the candidate singular points. The block size is also 16 × 16. If the neighborhood of a candidate singular point is a background or noisy region, the point detected by HT is considered as the final singular point. If the distance between the detected candidate core and delta is close enough and the Poincare indexes of them are both zero, the fingerprint is classified into arch, and the detected core and delta will be erased. Fig. 9 shows the detected singular points on arch by HT. In Figs. 9b, 9c, 9d, and 9e, a lighter block means a larger gray value.

For nonarch fingerprint images, we need to confirm whether it is whorl or twin or not. If a second core exists, a fingerprint image is classified into a whorl or twin. Therefore, for all nonarch fingerprints, we try to find whether a second core exists or not. Singular point is in the region where the curvature of the ridge line is much larger than any other regions [22], [23]. The ridge line is traced from the first detected core. In tracing, if the curvature of a point and its
Fig. 9. Arch. (a) Original image. (b) Detected core gray image by HT for (a). (c) Detected core gray image overlap upon (a). (d) Detected delta gray image by HT for (a). (e) Detected delta gray image overlap upon (a). (f) Final output image.

Fig. 10. Distance of two cores is far enough. (a) Original image. (b) Detected core gray image by HT for (a). (c) Detected core gray image overlap upon (a). (d) Detected delta gray image by HT for (a). (e) Detected delta gray image overlap upon (a). (f) Detected core (o) and delta (Δ), white line is trace line.
Fig. 11. Distance of two cores is fairly close. (a) Original image. (b) Detected core gray image by HT for (a). (c) Detected core gray image overlap upon (a). (d) Detected delta gray image by HT for (a). (e) Detected delta gray image overlap upon (a). (f) Detected core (o) and delta (Δ), white line is trace line.

Fig. 12. Loop. (a) Original image. (b) Detected core gray image by HT for (a). (c) Detected core gray image overlap upon (a). (d) Detected delta gray image by HT for (a). (e) Detected delta gray image overlap upon (a). (f) Detected core (o) and delta (Δ), white line is trace line.
consecutive three points are all larger than a threshold, it is considered as a second candidate core. Then, the Poincare index is also used to confirm and refine the position of the second candidate core. If the second core is confirmed, a fingerprint is classified into whorl or twin. In a fingerprint, core and delta exist in pair(s) [21]. Positions of cores are higher than deltas and in the middle of deltas [21]. Now, a second delta exists in a fingerprint and needs to be found. As two cores and a delta have been detected, their positions will be used to find the second delta. Assume the positions of the detected two cores and one delta are \((x_{c1}, y_{c1}), (x_{c2}, y_{c2}),\) and \((x_{d1}, y_{d1}).\) The length and height of the fingerprint image are represented as \(l\) and \(h\), respectively. If \(x_{d1} > \max(x_{c1}, x_{c2})\), the second delta is in area \([0, \min(x_{c1}, x_{c2})] \times [\max(y_{c1}, y_{c2}), h].\) If \(x_{d1} < \min(x_{c1}, x_{c2})\), the second delta is in area \([\max(x_{c1}, x_{c2}), l] \times [\max(y_{c1}, y_{c2}), h].\) HT is implemented in this small area, and the second candidate delta can be found. Then, it will be verified and precisely located by Poincare index. If ridge line is traced long enough, and no large curvature region is found, the fingerprint image is classified into tent arch or loop, as Fig. 12 shows. Table 1 shows summary of our algorithm to detect singular points in real fingerprints.

### Table 1
Summary of Our Algorithm to Detect Singular Points

<table>
<thead>
<tr>
<th>Input:</th>
<th>Segmented orientation image.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output:</td>
<td>The detected singular points.</td>
</tr>
<tr>
<td>Step 1:</td>
<td>HT detects a candidate core and delta. Go to step 2.</td>
</tr>
<tr>
<td>Step 2:</td>
<td>Use Poincare index and distance of the detected core and delta to judge arch. If fingerprint is arch, erased the singular points. The algorithm ends and go to Output. Else, go to step 3.</td>
</tr>
<tr>
<td>Step 3:</td>
<td>Ridge tracing try to find a second core. If a second core exists, the fingerprint is whorl or twin. Go to step 4. Else, the fingerprint is tent arch or loop. The algorithm ends and go to Output.</td>
</tr>
<tr>
<td>Step 4:</td>
<td>HT detects the second delta in a small area. The algorithm ends and go to Output.</td>
</tr>
</tbody>
</table>

In this experiment, the former 100 fingerprint images F0001-01 to F0100-10 in NIST-4 are used to make statistics. In these 100 fingerprint images, there are 30 arches, 4 tent arches, 48 loops, and 18 whorls (including twins). In other words, there are 88 cores and 88 deltas in these 100 fingerprint images. Some of these images are suffered from noise, as Fig. 13 shows. Arch has no singular point, tent arch or loop has one pair of singular points, and whorl or twin has two pairs of singular points. Our method can also classify fingerprint images into the above categories according to the pair(s) of detected singular point(s). Table 2 shows classification results by a confusion matrix. Except that two arches and a whorl are misclassified, other 97 fingerprint images are right classified. In our work, HT uses rather global information to detect singular points. This makes our algorithm fairly robust to noise. The comparison of our method with Poincare index is carried out. Table 3 shows a comparison of our method with the Poincare index. The results of comparison illustrate that our method has more excellent performance than Poincare index.

### Table 2
Classification Results by a Confusion Matrix

<table>
<thead>
<tr>
<th></th>
<th>arch</th>
<th>loop or tent arch</th>
<th>whorl or twin</th>
</tr>
</thead>
<tbody>
<tr>
<td>arch</td>
<td>28</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>loop or tent arch</td>
<td>0</td>
<td>52</td>
<td>0</td>
</tr>
<tr>
<td>whorl or twin</td>
<td>0</td>
<td>1</td>
<td>17</td>
</tr>
</tbody>
</table>

### Table 3
Comparison between Our Method and Poincare Index

<table>
<thead>
<tr>
<th>Method</th>
<th>block size</th>
<th>missed core</th>
<th>missed delta</th>
<th>spurious core</th>
<th>spurious delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>our method</td>
<td>16 x 16</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Poincare index</td>
<td>16 x 16</td>
<td>20</td>
<td>25</td>
<td>23</td>
<td>2</td>
</tr>
<tr>
<td>Poincare index</td>
<td>8 x 8</td>
<td>11</td>
<td>17</td>
<td>95</td>
<td>59</td>
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<tr>
<td>Poincare index</td>
<td>4 x 4</td>
<td>9</td>
<td>10</td>
<td>252</td>
<td>150</td>
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</tbody>
</table>

In our work, HT uses rather global information to detect singular points. This makes our algorithm fairly robust to noise. The comparison of our method with Poincare index is carried out. Table 3 shows a comparison of our method with the Poincare index. The results of comparison illustrate that our method has more excellent performance than Poincare index. For Poincare index, block orientation is also a dominant orientation computed by an orientation histogram in the level \(8 \times 8\) and \(4 \times 4\). Using our method, 87 pairs of singular points are detected, and one pair of singular points is missed. However, only using Poincare index to detect
singular points, the more local the information, the more that spurious singular points appear. Table 4 shows the statistical position error. Error in the results is distance in blocks between the detected singular points and human operator. Block size is also $16 \times 16$. In our work, if the error is more than two block distance, the detected singular point is misplaced. It is observed that our algorithm works satisfactorily, i.e., one core misplaced and three deltas misplaced in 100 fingerprints. Fig. 14 shows a comparison of results in Fig. 13 using our method and Poincare index, respectively.

5 CONCLUSION

This paper combines Zero-pole Model and HT to detect singular points. Orientation of singular points is defined on the basis of the Zero-pole Model. This can explain well why delta is the confluent point of three different flowlike directions. Based on the orientation field computed in Section 2, the detection of singular points is simplified to determine the parameters of the Zero-pole Model. In our work, HT uses rather global information of fingerprint images to detect singular points. HT is relatively unaffected by noise. This makes our algorithm more robust to noise than methods that only use local information. Because of the flaws of the Zero-pole Model, Poincare index is used to refine positions of candidate singular points. In a way, the results of singular points detection are limited by the accuracy of the Poincare index. In fact, global information combined with local information to detect singular points have good performance in practice. Experimental results show that our algorithm performs well and fast enough for real-time applications.

In our future work, we will rectify the Zero-pole Model. The rectified model will be easier to compute, and it will fit the actual fingerprint orientation perfectly. If so, local information may be not necessary to refine the positions of singular points. This will speedup our algorithm and make the algorithm more reliable to detect singular points in fingerprint images.

ACKNOWLEDGMENT

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<table>
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<th>0</th>
<th>1</th>
<th>2</th>
<th>&gt;2</th>
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</thead>
<tbody>
<tr>
<td>core</td>
<td>79</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>delta</td>
<td>80</td>
<td>3</td>
<td>1</td>
<td>3</td>
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</tbody>
</table>
References


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